

MULTIMEDIA



UNIVERSITY

STUDENT ID NO

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MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2019/2020

BST3254 – MONTE CARLO SIMULATION TECHNIQUES
(All sections / Groups)

28 FEBRUARY 2020
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of 6 pages excluding the cover page.
2. Attempt ALL FOUR questions.
3. Write your answers in the Answer Booklet provided.
4. The statistical tables are attached at the end of this question paper.

Question 1 (25 marks)

Consider the following historical data from an electrical company:

| Demand Data | | | | | |
|-------------|------|------|------|------|------|
| Demand | 5 | 12 | 20 | 24 | 30 |
| Probability | 0.10 | 0.35 | 0.15 | 0.20 | 0.20 |

| Lead time Data | | | | |
|-----------------------|------|------|------|------|
| Lead time (in months) | 1 | 2 | 3 | 4 |
| Probability | 0.25 | 0.17 | 0.36 | 0.22 |

The company orders 40 units of the product whenever the inventory level reaches 5 or less by the end of the month. The costs associated with managing inventory includes the cost of ordering, the carrying cost and the stockout cost. The cost of ordering of RM 50 per order. The carrying cost of RM 2 per month for each unit that is left in the inventory at the end of each year. The stockout cost has been set at RM 3 for every short unit.

(a) Establish the random number intervals for the demand and lead time data set. (4 marks)

(b) Simulate 12 months of operation for the company assuming that there is currently 20 units in inventory. Use the following random numbers for your simulation:

Demand: 29, 07, 24, 15, 45, 96, 39, 81, 76, 67, 55, 47

Lead: 40, 24, 32, 11, 72

(12 marks)

(c) Compute the average holding cost, average ordering cost and average stockout cost for the company.

(9 marks)

Continued...

Question 2 (25 marks)

- a) Model verification and validation are important activities in a simulation project. They determine the success of the project. However, these activities are often neglected when the simulation is carried out.

State **two** possible reasons a researcher would neglect such important activities.

(4 marks)

- b) Generate a sequence of two random integers and their corresponding random numbers (in four decimal places) using the mixed congruential method with $X_0 = 3$, $a = 8$, $c = 7$ and $m = 128$.

(4 marks)

- c) Test the following sequence of numbers, R_i , for independence. Let $i = 1$ and $m = 8$ lags.

| i-th | R_i |
|------|-------|
| 1 | 0.75 |
| 2 | 0.89 |
| 3 | 0.33 |
| 4 | 0.68 |
| 5 | 0.19 |
| 6 | 0.69 |
| 7 | 0.31 |
| 8 | 0.35 |
| 9 | 0.49 |
| 10 | 0.36 |

| i-th | R_i |
|------|-------|
| 11 | 0.12 |
| 12 | 0.64 |
| 13 | 0.91 |
| 14 | 0.05 |
| 15 | 0.93 |
| 16 | 0.01 |
| 17 | 0.28 |
| 18 | 0.41 |
| 19 | 0.43 |
| 20 | 0.88 |

| i-th | R_i |
|------|-------|
| 21 | 0.23 |
| 22 | 0.99 |
| 23 | 0.60 |
| 24 | 0.95 |
| 25 | 0.87 |
| 26 | 0.28 |
| 27 | 0.15 |
| 28 | 0.27 |
| 29 | 0.58 |
| 30 | 0.83 |

- (i) Find the largest integer M such that $i + (M + 1)m \leq N$. (4 marks)

- (ii) At significance level $\alpha = 0.05$, perform the autocorrelation test on the data above.

(13 marks)

Continued...

Question 3 (25 marks)

Consider the following pdf:

$$f(x) = \begin{cases} \frac{x^2 + x}{x} & -1 < x < 0 \\ \frac{x - x^2}{x}, & 0 \leq x < 1 \end{cases}$$

- a) Show that the random variate generator for the random variable X is as follows:

$$X = \begin{cases} -1 + \sqrt{2R}, & \text{for } 0 < R < 1/2 \\ 1 - \sqrt{-2R + 2}, & \text{for } 1/2 \leq R < 1 \end{cases}$$

(22 marks)

- b) Use the random variate generator developed in (a) to generate random variates that corresponds to the random numbers R_i given below:

0.136 0.500 0.789

(3 marks)

Question 4 (25 marks)

- a) Test the following sequence of random numbers for uniformity at significance level of 1%.

0.896 0.477 0.010 0.382 0.634

(10 marks)

- b) Consider the following input data for a simulation model that predicts the factory's damage costs.

| Number of accidents | Frequency | Probability, $P(X = x)$ |
|---------------------|-----------|-------------------------|
| 0 | 44 | 0.47237 |
| 1 | 33 | 0.35427 |
| 2 | 10 | 0.13285 |
| 3 | 4 | 0.03321 |
| 4 | 1 | 0.00621 |
| 5 or more | 0 | 0.00106 |

Apply the chi-square test to the sample data to test the hypothesis that the underlying distribution is Poisson at significance level $\alpha = 0.05$.

(15 marks)

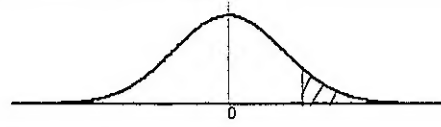
End of Questions.

APPENDIX – BST3254 MONTE CARLO SIMULATION TECHNIQUES

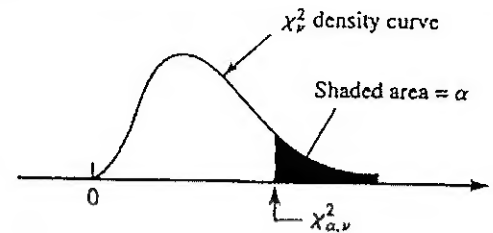
STATISTICAL TABLES

| Kolmogorov - Smirnov Critical Values | | | |
|--------------------------------------|-------------------------|-------------------------|-------------------------|
| Degrees of Freedom | | | |
| (N) | D _{0.10} | D _{0.05} | D _{0.01} |
| 1 | 0.950 | 0.975 | 0.995 |
| 2 | 0.776 | 0.842 | 0.929 |
| 3 | 0.642 | 0.708 | 0.828 |
| 4 | 0.564 | 0.624 | 0.733 |
| 5 | 0.510 | 0.565 | 0.669 |
| 6 | 0.470 | 0.521 | 0.618 |
| 7 | 0.438 | 0.486 | 0.577 |
| 8 | 0.411 | 0.457 | 0.543 |
| 9 | 0.388 | 0.432 | 0.514 |
| 10 | 0.368 | 0.410 | 0.490 |
| 11 | 0.352 | 0.391 | 0.468 |
| 12 | 0.338 | 0.375 | 0.450 |
| 13 | 0.325 | 0.361 | 0.433 |
| 14 | 0.314 | 0.349 | 0.418 |
| 15 | 0.304 | 0.338 | 0.404 |
| 16 | 0.295 | 0.328 | 0.392 |
| 17 | 0.286 | 0.318 | 0.381 |
| 18 | 0.278 | 0.309 | 0.371 |
| 19 | 0.272 | 0.301 | 0.363 |
| 20 | 0.264 | 0.294 | 0.356 |
| 25 | 0.240 | 0.270 | 0.320 |
| 30 | 0.220 | 0.240 | 0.290 |
| 35 | 0.210 | 0.230 | 0.270 |
| Over 35 | $\frac{1.22}{\sqrt{N}}$ | $\frac{1.36}{\sqrt{N}}$ | $\frac{1.63}{\sqrt{N}}$ |

Table 1
The Upper Tail Area Under the
Standard Normal Curve



| Z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.0 | 0.5000 | 0.4960 | 0.4920 | 0.4880 | 0.4840 | 0.4801 | 0.4761 | 0.4721 | 0.4681 | 0.4641 |
| 0.1 | 0.4602 | 0.4562 | 0.4522 | 0.4483 | 0.4443 | 0.4404 | 0.4364 | 0.4325 | 0.4286 | 0.4247 |
| 0.2 | 0.4207 | 0.4168 | 0.4129 | 0.4090 | 0.4052 | 0.4013 | 0.3974 | 0.3936 | 0.3897 | 0.3859 |
| 0.3 | 0.3821 | 0.3783 | 0.3745 | 0.3707 | 0.3669 | 0.3632 | 0.3594 | 0.3557 | 0.3520 | 0.3483 |
| 0.4 | 0.3446 | 0.3409 | 0.3372 | 0.3336 | 0.3300 | 0.3264 | 0.3228 | 0.3192 | 0.3156 | 0.3121 |
| 0.5 | 0.3085 | 0.3050 | 0.3015 | 0.2981 | 0.2946 | 0.2912 | 0.2877 | 0.2843 | 0.2810 | 0.2776 |
| 0.6 | 0.2743 | 0.2709 | 0.2676 | 0.2643 | 0.2611 | 0.2578 | 0.2546 | 0.2514 | 0.2483 | 0.2451 |
| 0.7 | 0.2420 | 0.2389 | 0.2358 | 0.2327 | 0.2296 | 0.2266 | 0.2236 | 0.2206 | 0.2177 | 0.2148 |
| 0.8 | 0.2119 | 0.2090 | 0.2061 | 0.2033 | 0.2005 | 0.1977 | 0.1949 | 0.1922 | 0.1894 | 0.1867 |
| 0.9 | 0.1841 | 0.1814 | 0.1788 | 0.1762 | 0.1736 | 0.1711 | 0.1685 | 0.1660 | 0.1635 | 0.1611 |
| 1.0 | 0.1587 | 0.1562 | 0.1539 | 0.1515 | 0.1492 | 0.1469 | 0.1446 | 0.1423 | 0.1401 | 0.1379 |
| 1.1 | 0.1357 | 0.1335 | 0.1314 | 0.1292 | 0.1271 | 0.1251 | 0.1230 | 0.1210 | 0.1190 | 0.1170 |
| 1.2 | 0.1151 | 0.1131 | 0.1112 | 0.1093 | 0.1075 | 0.1056 | 0.1038 | 0.1020 | 0.1003 | 0.0985 |
| 1.3 | 0.0968 | 0.0951 | 0.0934 | 0.0918 | 0.0901 | 0.0885 | 0.0869 | 0.0853 | 0.0838 | 0.0823 |
| 1.4 | 0.0808 | 0.0793 | 0.0778 | 0.0764 | 0.0749 | 0.0735 | 0.0721 | 0.0708 | 0.0694 | 0.0681 |
| 1.5 | 0.0668 | 0.0655 | 0.0643 | 0.0630 | 0.0618 | 0.0606 | 0.0594 | 0.0582 | 0.0571 | 0.0559 |
| 1.6 | 0.0548 | 0.0537 | 0.0526 | 0.0516 | 0.0505 | 0.0495 | 0.0485 | 0.0475 | 0.0465 | 0.0455 |
| 1.7 | 0.0446 | 0.0436 | 0.0427 | 0.0418 | 0.0409 | 0.0401 | 0.0392 | 0.0384 | 0.0375 | 0.0367 |
| 1.8 | 0.0359 | 0.0351 | 0.0344 | 0.0336 | 0.0329 | 0.0322 | 0.0314 | 0.0307 | 0.0301 | 0.0294 |
| 1.9 | 0.0287 | 0.0281 | 0.0274 | 0.0268 | 0.0262 | 0.0256 | 0.0250 | 0.0244 | 0.0239 | 0.0233 |
| 2.0 | 0.0228 | 0.0222 | 0.0217 | 0.0212 | 0.0207 | 0.0202 | 0.0197 | 0.0192 | 0.0188 | 0.0183 |
| 2.1 | 0.0179 | 0.0174 | 0.0170 | 0.0166 | 0.0162 | 0.0158 | 0.0154 | 0.0150 | 0.0146 | 0.0143 |
| 2.2 | 0.0139 | 0.0136 | 0.0132 | 0.0129 | 0.0125 | 0.0122 | 0.0119 | 0.0116 | 0.0113 | 0.0110 |
| 2.3 | 0.0107 | 0.0104 | 0.0102 | 0.0099 | 0.0096 | 0.0094 | 0.0091 | 0.0089 | 0.0087 | 0.0084 |
| 2.4 | 0.0082 | 0.0080 | 0.0078 | 0.0075 | 0.0073 | 0.0071 | 0.0069 | 0.0068 | 0.0066 | 0.0064 |
| 2.5 | 0.0062 | 0.0060 | 0.0059 | 0.0057 | 0.0055 | 0.0054 | 0.0052 | 0.0051 | 0.0049 | 0.0048 |
| 2.6 | 0.0047 | 0.0045 | 0.0044 | 0.0043 | 0.0041 | 0.0040 | 0.0039 | 0.0038 | 0.0037 | 0.0036 |
| 2.7 | 0.0035 | 0.0034 | 0.0033 | 0.0032 | 0.0031 | 0.0030 | 0.0029 | 0.0028 | 0.0027 | 0.0026 |
| 2.8 | 0.0026 | 0.0025 | 0.0024 | 0.0023 | 0.0023 | 0.0022 | 0.0021 | 0.0021 | 0.0020 | 0.0019 |
| 2.9 | 0.0019 | 0.0018 | 0.0018 | 0.0017 | 0.0016 | 0.0016 | 0.0015 | 0.0015 | 0.0014 | 0.0014 |
| 3.0 | 0.0013 | 0.0013 | 0.0013 | 0.0012 | 0.0012 | 0.0011 | 0.0011 | 0.0011 | 0.0010 | 0.0010 |
| 3.1 | 0.0010 | 0.0009 | 0.0009 | 0.0009 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0007 | 0.0007 |
| 3.2 | 0.0007 | 0.0007 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0005 | 0.0005 | 0.0005 |
| 3.3 | 0.0005 | 0.0005 | 0.0005 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0003 |
| 3.4 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0002 |
| 3.5 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 | 0.0002 |
| 3.6 | 0.0002 | 0.0002 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 |
| 3.7 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 |
| 3.8 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 |
| 3.9 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4.0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

Table A.7 Critical Values for Chi-Squared Distributions

| ν | .995 | .99 | .975 | .95 | .90 | .10 | .05 | .025 | .01 | .005 |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1 | 0.000 | 0.000 | 0.001 | 0.004 | 0.016 | 2.706 | 3.843 | 5.025 | 6.637 | 7.882 |
| 2 | 0.010 | 0.020 | 0.051 | 0.103 | 0.211 | 4.605 | 5.992 | 7.378 | 9.210 | 10.597 |
| 3 | 0.072 | 0.115 | 0.216 | 0.352 | 0.584 | 6.251 | 7.815 | 9.348 | 11.344 | 12.837 |
| 4 | 0.207 | 0.297 | 0.484 | 0.711 | 1.064 | 7.779 | 9.488 | 11.143 | 13.277 | 14.860 |
| 5 | 0.412 | 0.554 | 0.831 | 1.145 | 1.610 | 9.236 | 11.070 | 12.832 | 15.085 | 16.748 |
| 6 | 0.676 | 0.872 | 1.237 | 1.635 | 2.204 | 10.645 | 12.592 | 14.440 | 16.812 | 18.548 |
| 7 | 0.989 | 1.239 | 1.690 | 2.167 | 2.833 | 12.017 | 14.067 | 16.012 | 18.474 | 20.276 |
| 8 | 1.344 | 1.646 | 2.180 | 2.733 | 3.490 | 13.362 | 15.507 | 17.534 | 20.090 | 21.954 |
| 9 | 1.735 | 2.088 | 2.700 | 3.325 | 4.168 | 14.684 | 16.919 | 19.022 | 21.665 | 23.587 |
| 10 | 2.156 | 2.558 | 3.247 | 3.940 | 4.865 | 15.987 | 18.307 | 20.483 | 23.209 | 25.188 |
| 11 | 2.603 | 3.053 | 3.816 | 4.575 | 5.578 | 17.275 | 19.675 | 21.920 | 24.724 | 26.755 |
| 12 | 3.074 | 3.571 | 4.404 | 5.226 | 6.304 | 18.549 | 21.026 | 23.337 | 26.217 | 28.300 |
| 13 | 3.565 | 4.107 | 5.009 | 5.892 | 7.041 | 19.812 | 22.362 | 24.735 | 27.687 | 29.817 |
| 14 | 4.075 | 4.660 | 5.629 | 6.571 | 7.790 | 21.064 | 23.685 | 26.119 | 29.141 | 31.319 |
| 15 | 4.600 | 5.229 | 6.262 | 7.261 | 8.547 | 22.307 | 24.996 | 27.488 | 30.577 | 32.799 |
| 16 | 5.142 | 5.812 | 6.908 | 7.962 | 9.312 | 23.542 | 26.296 | 28.845 | 32.000 | 34.267 |
| 17 | 5.697 | 6.407 | 7.564 | 8.682 | 10.085 | 24.769 | 27.587 | 30.190 | 33.408 | 35.716 |
| 18 | 6.265 | 7.015 | 8.231 | 9.390 | 10.865 | 25.989 | 28.869 | 31.526 | 34.805 | 37.156 |
| 19 | 6.843 | 7.632 | 8.906 | 10.117 | 11.651 | 27.203 | 30.143 | 32.852 | 36.190 | 38.580 |
| 20 | 7.434 | 8.260 | 9.591 | 10.851 | 12.443 | 28.412 | 31.410 | 34.170 | 37.566 | 39.997 |
| 21 | 8.033 | 8.897 | 10.283 | 11.591 | 13.240 | 29.615 | 32.670 | 35.478 | 38.930 | 41.399 |
| 22 | 8.643 | 9.542 | 10.982 | 12.338 | 14.042 | 30.813 | 33.924 | 36.781 | 40.289 | 42.796 |
| 23 | 9.260 | 10.195 | 11.688 | 13.090 | 14.848 | 32.007 | 35.172 | 38.075 | 41.637 | 44.179 |
| 24 | 9.886 | 10.856 | 12.401 | 13.848 | 15.659 | 33.196 | 36.415 | 39.364 | 42.980 | 45.558 |
| 25 | 10.519 | 11.523 | 13.120 | 14.611 | 16.473 | 34.381 | 37.652 | 40.646 | 44.313 | 46.925 |
| 26 | 11.160 | 12.198 | 13.844 | 15.379 | 17.292 | 35.563 | 38.885 | 41.923 | 45.642 | 48.290 |
| 27 | 11.807 | 12.878 | 14.573 | 16.151 | 18.114 | 36.741 | 40.113 | 43.194 | 46.962 | 49.642 |
| 28 | 12.461 | 13.565 | 15.308 | 16.928 | 18.939 | 37.916 | 41.337 | 44.461 | 48.278 | 50.993 |
| 29 | 13.120 | 14.256 | 16.147 | 17.708 | 19.768 | 39.087 | 42.557 | 45.772 | 49.586 | 52.333 |
| 30 | 13.787 | 14.954 | 16.791 | 18.493 | 20.599 | 40.256 | 43.773 | 46.979 | 50.892 | 53.672 |
| 31 | 14.457 | 15.655 | 17.538 | 19.280 | 21.433 | 41.422 | 44.985 | 48.231 | 52.190 | 55.000 |
| 32 | 15.134 | 16.362 | 18.291 | 20.072 | 22.271 | 42.585 | 46.194 | 49.480 | 53.486 | 56.328 |
| 33 | 15.814 | 17.073 | 19.046 | 20.866 | 23.110 | 43.745 | 47.400 | 50.724 | 54.774 | 57.646 |
| 34 | 16.501 | 17.789 | 19.806 | 21.664 | 23.952 | 44.903 | 48.602 | 51.966 | 56.061 | 58.964 |
| 35 | 17.191 | 18.508 | 20.569 | 22.465 | 24.796 | 46.059 | 49.802 | 53.203 | 57.340 | 60.272 |
| 36 | 17.887 | 19.233 | 21.336 | 23.269 | 25.643 | 47.212 | 50.998 | 54.437 | 58.619 | 61.581 |
| 37 | 18.584 | 19.960 | 22.105 | 24.075 | 26.492 | 48.363 | 52.192 | 55.667 | 59.891 | 62.880 |
| 38 | 19.289 | 20.691 | 22.878 | 24.884 | 27.343 | 49.513 | 53.384 | 56.896 | 61.162 | 64.181 |
| 39 | 19.994 | 21.425 | 23.654 | 25.695 | 28.196 | 50.660 | 54.572 | 58.119 | 62.426 | 65.473 |
| 40 | 20.706 | 22.164 | 24.433 | 26.509 | 29.050 | 51.805 | 55.758 | 59.342 | 63.691 | 66.766 |

For $\nu > 40$, $\chi^2_{\alpha, \nu} \approx \nu \left(1 - \frac{2}{9\nu} + z_{\alpha} \sqrt{\frac{2}{9\nu}} \right)^3$

